### **Presentations of Groups**



# **Groups with presentations in EDTOL**

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**L-presentations.** A group G with generating set S has a finite L-presentation if there are two finite sets of relations  $Q, R \subseteq \mathbb{F}_S$  and a finite set  $\Phi$  of endomorphisms of  $\mathbb{F}_S$  such that G is isomorphic to the quotient

 $\mathbb{F}_S/\langle\langle Q \cup \bigcup \phi(R)\rangle\rangle$ 

The Grigorchuk group is a special case of:

Theorem [Bartholdi 2002]

A finitely generated contracting self-similar regular branch group has an L-presentation.

Computation of nilpotent quotients is possible for L-presentations:

### Theorem [Barth., Eick, Hartung]

There is an algorithm that, given an Lpresentation for a group G and  $i \in \mathbb{N}$ , computes a finite presentation for the group  $G/\gamma_i(G)$ .

Computation of finite quotients yields:

Theorem [Hartung, 2011]

There is a Todd-Coxeter coset enumeration algorithm for groups with L-presentations.

# Results

# Theorem A

A group has an L-presentation if and only if it has an EDT0L presentation.

Remark: having a L-presentation is not independent of a generating family, while having an EDT0L presentation is. The new approach is more robust.

# Proposition B

Equality of marked groups is not semi-decidable for groups given by EDT0L presentations.

Let  $\mathcal{C}$  be the set of abelian-by-nilpotent groups, or that of groups that are virtual direct products of finitely many hyperbolic groups.

#### Theorem C

There is an algorithm that takes as input an EDT0L presentation of a marked group and a finite presentation for group in  $\mathcal{C}$ , and decides whether or not the latter is a marked quotient of the former.

Idea: starting from Bartholdi, Eick, Hartung replace notherianity by equational **notherianity**, and solvability of the word problem by **solvability of the universal** theory. We can then use recent results of Ciobanu, Holt, Rees.

#### Theorem D

The exists a residually nilpotent finitely generated group H with solvable word problem but uncomputable nilpotent quotients: no algorithm can, on input n, produce a finite presentation of  $H/\gamma_n(H)$ .

As a corollary, this group does not have an EDT0L presentation.

Groups given by presentations



#### References

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[3] LYSENOK, I. G. A system of defining relations for a grigorchuk group. Mathematical Notes of the Academy of Sciences of the USSR 38, 4 (10 1985), 784-792.